170 – Test example problems
CH8,9,10,11,12,13

**WARNING**: these are simply examples that showed up in previous semesters’ test. It does **NOT** mean that similar problems will be present in **THIS** semester’s test. Hence, you are suggested to use these problems to deepen your understanding on the CONCEPTS involved in these problems. Please do not waste your time memorize them.

**Q1: 9 points** - Given two masses \( m_1 \) (6 kg) and \( m_2 \) (15 kg), friction on the table \((\mu_s = 0.3, \mu_k = 0.2)\), and 3 pulleys of masses \( m_{p1}, m_{p2}, \) and \( m_{p3} \), the system is initially in an equilibrium state as the mass \( m_1 \) is held into place by an external hand (not drawn).

*It is clear that at the instant the hand is released, the system will not remain at equilibrium.*

a) If the mass \( m_1 \) is released from rest, what is its instantaneous acceleration if all pulley masses are negligible? **(3 points)**

b) If the mass \( m_1 \) is released from rest, what is its instantaneous acceleration if the pulley masses \( m_{p2}, \) and \( m_{p3} \) are negligible, but \( m_{p1} = 3 \) kg and has a non-negligible radius of 0.5m \((I_{pulley} = \frac{1}{2} mR^2)\)? **(3 points)**

c) If the mass \( m_1 \) is released from rest, what is its velocity after it has dropped a distance \( d \) of 0.5m if the pulley masses \( m_{p2}, \) and \( m_{p3} \) are negligible, but \( m_{p1} = 3 \) kg and has a non-negligible radius of 0.5m \((I_{pulley} = \frac{1}{2} mR^2)\)? **(3 points)**

**Bonus question (+2):** would any of your answer be different if the pulleys were 10 times bigger \((R \rightarrow 10*R)\)?
Q2: 6 points – This is a disk that can rotate freely about its axle at point A, located at this center. It is also at point A that the disk is mounted on its vertical support with a bearing (which implies that at point A, there are two forces from the vertical support acting on the disk to prevent any translation). The horizontal force F has a magnitude of 10N; the pulley has a mass of 0.5 kg and a radius of 0.5m ($I_{disk} = \frac{1}{2} mR^2$); friction is negligible.

a) What is the angular acceleration of the disk due to force F? (2 points)
b) What is the angular velocity after 5 seconds if the disk starts from rest? (2 points)
c) What are the direction and magnitude of the support forces acting on the disk from the vertical support? (2 points)
**Q3: 6 points** - The object consists of a homogeneous rod of length 2 meters, of mass 2kg welded to a 3kg homogeneous sphere of radius R=0.5m.

The moment of inertia of a bar rotating about an axis going through its center of mass (perpendicular to the page) is: \( I = \frac{1}{12} mL^2 \) (\( L \): length of the bar)

The moment of inertia of a disk rotating about an axis going through its center of mass (perpendicular to the page) is: \( I = \frac{2}{5} mR^2 \) (\( R \): radius of the disk)

Center of mass: \( x = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} \)

a) Calculate the position of the center of mass of the object with respect to point A (2 points)
b) Calculate the total moment of inertia of the pendulum about an axis perpendicular to the page going through its center of mass. (4 points)
Q4: 9 points - An object of mass $m$ (5 kg) is dropped from rest at a height of 2 meters above the horizontal bar of length of 3 meters, which is also at rest ($\omega = 0 \text{s}^{-1}$) – the distance $d$ is 0.5m. Upon collision, the mass $m$ sticks to the horizontal bar and both rotate about the fixed axis of rotation point A.

By which angle $\theta$ is the bar/mass system is going to rotate such that they will reach zero velocity again (consider the mass $m$ as a point particle in your angular momentum analysis; $I_{\text{bar}} = \frac{1}{3} mL^2$). (Due to the way this system is set up, your angle will be negative, don’t worry and keep its absolute value).
Q5: 6 points – Imagine that we are able to take a snapshot of the 2 electrons e1, and e2, located at the instantaneous respective distances r1 and r2 around a helium nucleus composed of 2 protons and 1 neutron.

\[ m_{e1} = m_{e2} = 9 \times 10^{-31} \, \text{kg}, \quad m_{\text{proton}} = m_{\text{neutron}} = 1.6 \times 10^{-27} \, \text{kg}, \quad r_1 = 1 \times 10^{-10} \, \text{m}, \]

\[ r_2 = 1.5 \times 10^{-10} \, \text{m}, \quad \alpha = 30^\circ \]

a) Use the superposition principle to determine the magnitude and direction of the gravitational force due to both electrons on the nucleus at this instant. (4 points)

b) Based on the magnitude of the value obtained in a), and comparing it to the E&M force due to these two electrons and the nucleus, which is \(10^{-2} \, \text{N}\), do we really care about the gravitational effect within a helium atom? (2 points)

(6 points) Q1: Two masses \( m_A \) (10kg) and \( m_B \) (5kg) are hanging happily over two pulleys: on the right (next to \( m_B \)) a massive (\( m = 2 \, \text{kg} \)) and frictionless pulleys of radius \( R \) (0.5m); on the left (next to \( m_A \)) a massless and frictionless pulleys of same radius \( R \) (0.5m). Both are connected to a non-stretchable string – no friction is considered in this problem. If released from rest, object \( m_A \) will drop by a distance \( h \) (2.5 meters). After the system has dropped by this distance \( h \), calculate (show all your steps):

1. The velocity of block
2. The acceleration of block

Make sure to show ALL your coordinate systems
**Q2:** A horizontal homogenous bar of length $L$ (5m) and mass $m$ (10kg) is at translational equilibrium, and positioned about its center of mass on a pivot. The poor bar is subjected to two forces $F_1 = 5N$ (at $\theta = 30^\circ$), acting at a point located at a fourth of the length of the bar as measured from the left end, and $F_2$. If the bar is spinning clockwise with an angular acceleration of $0.5s^{-2}$, calculate the magnitude of $F_2$, and the resultant force(s) from the pivot into the bar.

![Diagram of the bar and forces](image)

**Q3:** Tarzan ($m_r = 80kg$) and Jane ($m_j = 60kg$) are watching the stars at rest. They then decide to take a swing with a rope of length ($L = 50m$) and mass ($m_r = 20kg$) from an angle ($\theta = 45^\circ$). However, as the rope is vertical (bottom of the swing), Tarzan loses the grip of Jane and goes flying horizontally with a linear velocity of 7 m/s. At what angle $\alpha$ will Jane swing back up (where her velocity will be zero)? (consider Jane and Tarzan as point particles).

![Diagram of the swing](image)

**Q4:** A mass $m$ (4kg) is attached to a spring ($k = 100N/m$). The spring is compressed by a distance $d$ of 0.5m. At the instant of release, the mass attached to the spring is given a linear velocity of $v = 2ms^{-1}$ towards the right. Derive the differential equation modeling SHO using either Newton’s method of Energy (Hamiltonian), and obtain the specific equation describing the position of the mass attached to the spring as a function of time (i.e. obtain: $x(t)$). The solution of a Simple Harmonic Oscillator (SHO) differential equation is: $x(t) = A\cos(\omega t) + B\sin(\omega t)$. 

![Diagram of the spring and mass](image)
(3 points) Q5: What is the derivative of the translational kinetic energy with respect to velocity? i.e. what is: \( \frac{d}{dv}(KE) \). To what other concepts in physics is your answer related to? Predict what the derivative of the rotational kinetic energy with respect to angular velocity would be (this is a problem that Bud worked out in one of the Al Bondy episodes).

(5 points) Q6: A ball of mass \( m = 60 \text{kg} \) is released from a height of 2m and with an initial velocity of \( v = 2 \text{ms}^{-1} \) and an angle of \( \theta = 30^\circ \). During the collision between the floor and the ball, is the linear momentum of the ball conserved? (1 point) (watch out, it’s a trick question). If the ball bounces back at the same height that it was original released, what is the energy lost (2 points) during collision between the floor and the ball, and what is the value of the coefficient of restitution (2 points) defined as: \( e = \frac{V_{Af} - V_{Bf}}{V_{Bi} - V_{Ai}} \)?
(6 points) Q8: Multiple questions (circle the correct answer):

Four uniform objects having the same mass and diameter are released simultaneously from rest at the same distance above the bottom of a hill and roll down without slipping. Which of these objects will be the first one to reach the bottom of the hill? (2 points)

- A the solid sphere
- B the solid cylinder
- C the thin-walled hollow cylinder
- D the thin-walled hollow sphere

Two uniform solid spheres of the same size, but different mass, are released from rest simultaneously at the same height on a hill and roll. (2 points)

Which of the following statement(s) about these spheres are true?

- A Both spheres reach the bottom at the same time.
- B The heavier sphere reaches the bottom ahead of the lighter one.
- C Both spheres arrive at the bottom with the same forward speed.
- D Both spheres arrive at the bottom with the same total kinetic energy.

A thin uniform bar has a moment of inertia I about an axis perpendicular to it through its center. If both the mass and length of this bar are doubled, its moment of inertia about the same axis will be: (2 points)

- A 2I
- B 4I
- C 8I
- D 16I

4 points) Q9: 5 forces act on a truss (bridge) built by civil engineering students as part of a design project. The dimensions are b=300m, h=400m, and F=100N. If the bridge is at equilibrium, and if we consider the truss massless, calculate the forces at point B.

Bonus points (4 points): obtain the forces Ax and Ay at point A.
A mass $m$ (5kg) is attached to a pendulum (of length $L = 1.5m$). At the instant of release, the pendulum is in its equilibrium position and is given an angular velocity of $\dot{\theta} = 2s^{-1}$ and an angular acceleration of $\ddot{\theta} = 0.5s^{-1}$. Obtain the specific equation describing the angular position of the pendulum as a function of time (i.e. obtain: $\theta(t)$).

The solution of a Simple Harmonic Oscillator (SHO) differential equation is: $\theta(t) = A\cos(\omega t) + B\sin(\omega t)$.

A solid sphere ball (mass=1kg, Radius=0.2m) is rolling down an incline plane from rest and an initial height of 2 meters. As it reaches the bottom of the slope, it hits a pendulum* (this pendulum is made of a string (length=2m, mass of 2kg) attached to a disk of radius $R=0.5m$, and of mass 3 kg) which after impact, swings up to an unknown angle. The rolling ball keeps rolling in the horizontal direction with a velocity of 2 m/s after the impact with the pendulum.

Obtain the velocity of the rolling ball at the bottom of the slope (2 points):
Obtain the velocity of the pendulum ball right after impact in the horizontal direction. (2 points)
Obtain the angle through which the pendulum will swing, due to impact with the solid sphere. (2 points)
Was the collision between the solid sphere and pendulum elastic? If not, give the amount of energy that was lost during the collision. (2 points)

A ladder of length $L=12m$ and mass $m=45kg$ leans against a slick wall (that is, there is no friction between the ladder and the wall). The ladder’s upper end is at height $h=9.3m$ above the pavement on which the lower end is supported (the pavement is not frictionless). The ladder’s center of mass is $L/3$ from the lower end, along the length of the ladder. A firefighter of mass $M=72kg$ climbs the ladder until her center of mass is $L/2$ from the lower end.

What then is the magnitude of the force on the ladder from the wall?
What then is the magnitude of the force on the ladder from the pavement?

The beam is uniform and weighs 60N. If $W=200N$, find the tension in the tie rope and the x and y component of the force that the hinge exerts on the wall (no, you don’t need the value of $L$...;)
The uniform bar shown below weighs 40N and is subjected to the forces shown. Find the magnitude, location, and direction of the force needed to keep the bar in equilibrium.

Two masses $m_A$ (4kg) and $m_B$ (20kg) are hanging happily over a massive and frictionless pulley ($m=2kg$) of radius $R$ (0.5m). Both are connected to a non-stretchable string – no friction is considered in this problem. If released from rest (with a spring initially unstretched), object $m_A$ will rise by a distance $h$ (0.5 meters). Using $k=100N/m$, and after the system has moved by this distance $h$, calculate (show all your steps):

3. The velocity of each block
4. The acceleration of each block
A mass of 5 kg is attached to a massive pulley (2kg) of radius of 100mm and is released from rest. What is the angular acceleration of the system? What is the normal force from the axel into the wheel if this system is in translational equilibrium?
The 400 kg satellite S traveling at 7 km/s is hit by a 1 kg meteor M traveling at 12 km/s. The meteor is embedded in the satellite by the impact. Determine the magnitude of the velocity after impact and the value of beta (indicated in the diagram below).

(4 points) Q5: Let's consider the soccer ball in the figure below. The ball has mass 0.4 kg. Initially it moves horizontally to the left at 20 m/s, but then it is kicked and given a velocity with magnitude 30 m/s and direction 45 degrees upward and to the right.

(a) Before-and-after diagram

Find the impulse of the force in the x and in the y direction separately (hint: use the change of momentum! And be careful on your momentum directions!)

ANSWER: ________________________________

Calculate the magnitude of the overall force on the ball, assuming a collision time of 0.01s
(9 points) Q6: You are facing a variation of your favorite atwood system below, which is initially at rest: (m1=5kg, m2=2kg, mpulley=2kg, coefficient of static friction=0.4, and coefficient of kinetic friction=0.2).

Find the acceleration of the system

ANSWER: ____________________________________________________

Find the velocity of the block m2 after it has fallen 2 meters

ANSWER: ____________________________________________________

Find the velocity of the block m2 after it has fallen for 2 seconds.

ANSWER: ____________________________________________________

(6 points) Q7: The graph below closely approximates the displacement x of a tuning fork as a function of time t as it is playing a single note.
Based on this figure, obtain:

a) The amplitude (1 point)
   ANSWER: ____________________________________________________

b) The period (1 point)
   ANSWER: ____________________________________________________

c) The frequency (1 point)
   ANSWER: ____________________________________________________

d) The angular frequency (1 point)
   ANSWER: ____________________________________________________

e) Write down mathematically the position vs time of this tuning fork (1 point)
   ANSWER: ____________________________________________________

d) The maximum acceleration (1 point)
   ANSWER: ____________________________________________________
(2 points) \(Q8:\) What is the acceleration of the earth due to the sun? (mass of the sun=2\(\times\)10\(^{30}\) kg, distance earth/sun is 150\(\times\)10\(^{6}\) km)

ANSWER: ____________________________________________________